Pre Calc
Ch. 2 test review
Solutions

① $x^2 - 6x + 4 = 0$
$-4 \quad -4$

$x^2 - 6x = -4$
$+9 \quad +9$

$x^2 - 6x + 9 = 5$

$(x-3)^2 = 5$

$x - 3 = \pm\sqrt{5}$
$+3 \quad +3$

$\boxed{x = 3 \pm \sqrt{5}}$

② $\frac{3x^2 + 30x + 81}{3} = \frac{0}{3}$

$x^2 + 10x + 27 = 0$
$-27 \quad -27$

$x^2 + 10x = -27$
$+25 \quad +25$

$x^2 + 10x + 25 = -2$

$(x+5)^2 = -2$

$x + 5 = \pm\sqrt{-2}$
$-5 \quad -5$

$\boxed{x = -5 \pm i\sqrt{2}}$

③ $-3x^2 - 8x + 14 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{8 \pm \sqrt{64 + 168}}{-6}$

$x = \frac{8 \pm \sqrt{232}}{-6} = \frac{8 \pm 2\sqrt{58}}{-6}$

$\boxed{x = \frac{4 \pm \sqrt{58}}{-3}}$

④ $2x^2 - 6x + 11 = 0$

$x = \frac{6 \pm \sqrt{36 - 88}}{4}$

$x = \frac{6 \pm \sqrt{-52}}{4}$

$x = \frac{6 \pm 2i\sqrt{13}}{4}$

$\boxed{x = \frac{3 \pm i\sqrt{13}}{2}}$

⑤ $g(x) = x^2 - 8x + 14$

$g(x) = x^2 - 8x + 16 - 2$

$g(x) = (x-4)^2 - 2$

Vertex: $(4, -2)$

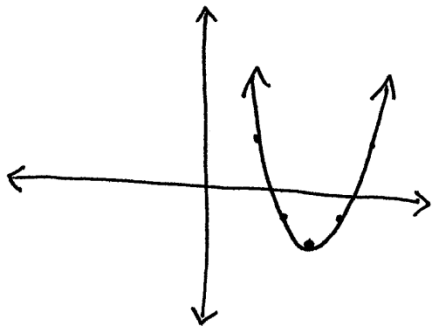
axis of sym: $x = 4$

roots: $(x-4)^2 - 2 = 0$

$(x-4)^2 = 2$

$x - 4 = \pm\sqrt{2}$

$\boxed{x = 4 \pm \sqrt{2}}$



Pattern: $\frac{1}{1}, \frac{3}{1}, \frac{5}{1}$

⑥ $h(x) = 3x^2 + 12x + 7$

Vertex: $x = \frac{-b}{2a} = \frac{-12}{6} = -2$

$y = 3(-2)^2 + 12(-2) + 7$

$y = 12 - 24 + 7 = -5$

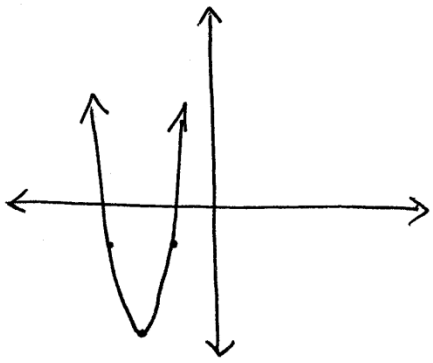
$(-2, -5)$

axis of sym: $x = -2$

roots: $x = \frac{-12 \pm \sqrt{144 - 84}}{6} = \frac{-12 \pm \sqrt{60}}{6}$

$\frac{-12 \pm 2\sqrt{15}}{6} = \boxed{\frac{-6 \pm \sqrt{15}}{3}}$

Pattern: $\frac{3}{1}, \frac{9}{1}, \frac{15}{1}$

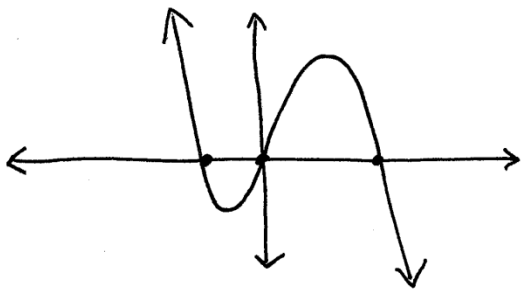


⑦ $-4x^3 + 4x^2 + 15x = 0$

$-x(4x^2 - 4x - 15) = 0$

$-x(2x+3)(2x-5) = 0$

$x = 0 \quad x = -\frac{3}{2} \quad x = \frac{5}{2}$



*Neg leading coefficient on a 3rd degree polynomial:

$x \to \infty, \; y \to -\infty$

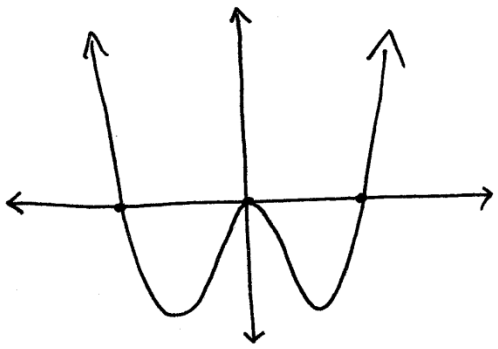
$x \to -\infty, \; y \to \infty$

⑧ $x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x^2(x+2)(x-2) = 0$

$x = -2, 2, 0$ (mult of 2)



* 4th degree polynomial with positive leading coefficient:

$x \to \infty, \; y \to \infty$

$x \to -\infty, \; y \to -\infty$

⑨ $x = -2, \sqrt{3}, -\sqrt{3}$

$f(x) = (x+2)(x-\sqrt{3})(x+\sqrt{3})$

$f(x) = (x+2)(x^2-3)$

$\boxed{f(x) = x^3+2x^2-3x-6}$

⑩ $x=0, \quad x=-2, \quad x=3 \text{ (mult 2)}$

$f(x) = x(x+2)(x-3)(x-3)$

$f(x) = (x^2+2x)(x^2-6x+9)$

$f(x) = x^4-6x^3+9x^2+2x^3-12x^2+18x$

$\boxed{f(x) = x^4-4x^3-3x^2+18x}$

⑪ $x^5-x^4-15x^3+9x^2+54x$

$x(x^4-x^3-15x^2+9x+54)$

3 | 1 −1 −15 9 54

1	2	−9	−18	0

−3 | 1 2 −9 −18

1	−1	−6	0

$x(x+3)(x-3)(x^2-x-6) = 0$

$x(x+3)(x-3)(x+2)(x-3) = 0$

$\boxed{x=0 \quad x=-3 \quad x=-2 \quad x=3 \text{ (mult of 2)}}$

* Since there is an x in every term, $x=0$ is a root.

* Since x^2-9 is a factor, there are roots at $x=3$ and $x=-3$

$x(x+3)(x-3)(\quad)$ to get the other factor, use synthetic division.

(12) $\dfrac{3-2i}{7+3i}\ \dfrac{(7-3i)}{(7-3i)} = \dfrac{21-9i-14i+6i^2}{49-9i^2} =$

$\dfrac{21-23i+6(-1)}{49-9(-1)} = \boxed{\dfrac{15-23i}{58}}$

(13) $x = 3, 5i, -5i, 0$

$f(x) = (x-3)(x-5i)(x+5i)(x)$

$(x-3)(x^2-5i^2)(x)$

$(x-3)(x^2+5)(x)$

$(x^3-3x^2+5x-15)(x)$

$\boxed{f(x) = x^4-3x^3+5x^2-15x}$

(14) $f(x) = x^4 + 6x^3 - x^2 - 54x - 72$

Possible Rational Roots:

$\pm(1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72)$

x	1	6	−1	−54	−72
−1	1	5	−6	−48	−24
−2	1	4	−9	−36	0

$f(x) = + \; +\!\searrow\!- \; - \; -$ ①

$f(-x) = +\!\searrow\!- \; -\!\searrow\!+ \; -\!\searrow\!-$ ③

try neg roots

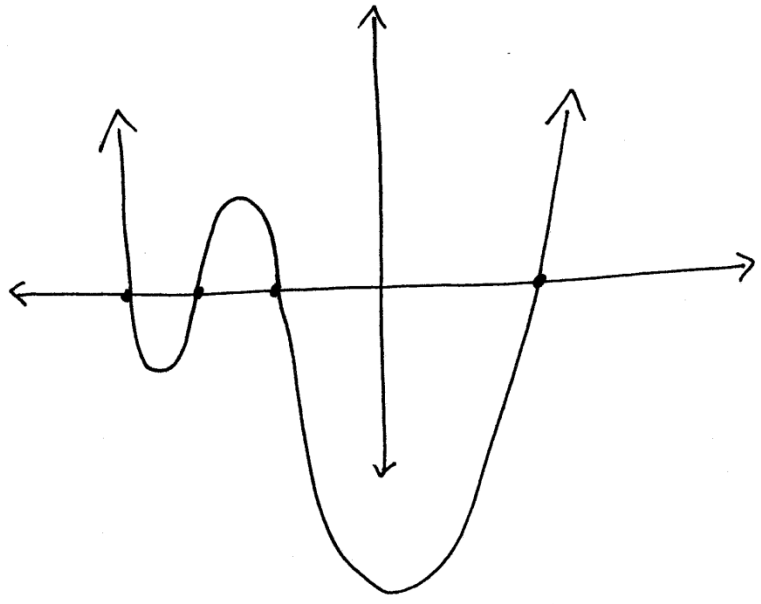
$(x+2)(x^3 + 4x^2 - 9x - 36)$

x	1	4	−9	−36
−3	1	1	−12	0

$(x+2)(x+3)(x^2 + x - 12) = 0$

$(x+2)(x+3)(x+4)(x-3) = 0$

$x = -2, -3, -4, 3$



(15) $f(x) = 2x^3 + 13x^2 - 13x + 3$

p: $\pm(1, 3)$

q: $\pm(1, 2)$

Possible Rational Roots:

$\pm\left(1, 3, \frac{1}{2}, \frac{3}{2}\right)$

X	2	13	-13	3
1	2	15	2	5
3	2	19	44	135
$\frac{1}{2}$	2	14	-6	0

$f(x) = + \; + \; - \; + \;$ ②

$f(-x) = - \; + \; + \; +$ ①

try pos roots

$\left(x - \frac{1}{2}\right)(2x^2 + 14x - 6) = 0$

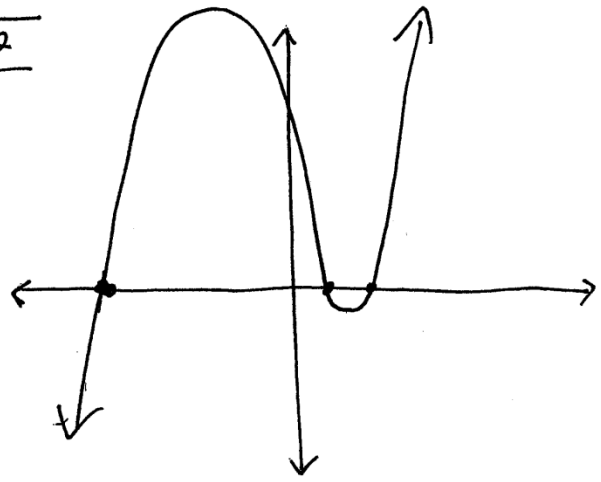
$2\left(x - \frac{1}{2}\right)(x^2 + 7x - 3) = 0$

$2\left(x - \frac{1}{2}\right)(x + \;)(x - \;) = 0$

cant be factored:

$x = \frac{1}{2} \quad x = \frac{-7 \pm \sqrt{49 + 12}}{2}$

$x = \frac{1}{2} \quad x = \frac{-7 \pm \sqrt{61}}{2}$

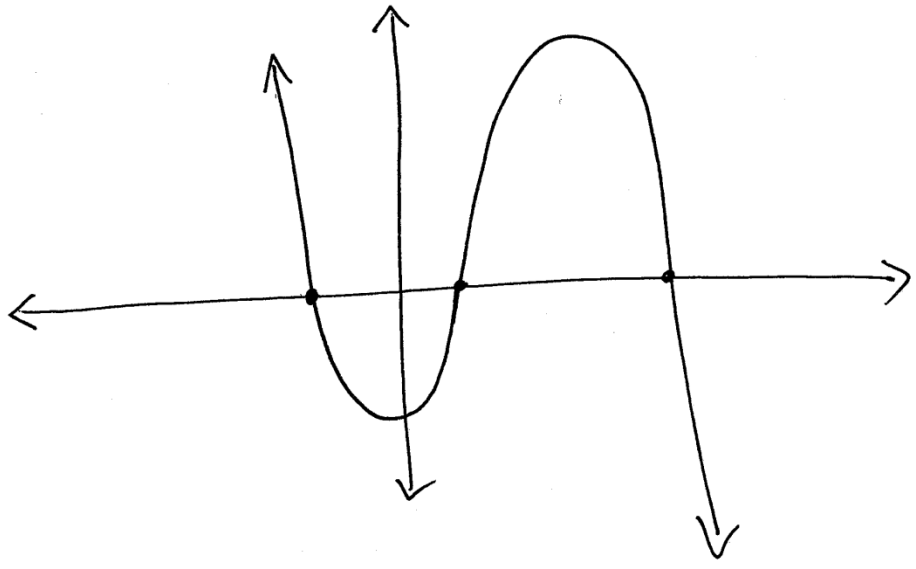


(16) $f(x) = -2x^3 + 5x^2 + 9x - 8$

X	-2	5	9	-8
-3	-2	11	-24	64
Lower Bound (-2)	-2	9	-9	10
root → -1	-2	7	2	-10
0	-2	5	9	-8
root → 1	-2	3	12	4
2	-2	1	11	14
3	-2	-1	6	10
root → Upper Bound (4)	-2	-3	-3	-20

Roots:

$X \approx -1.5, .7, 3.3$



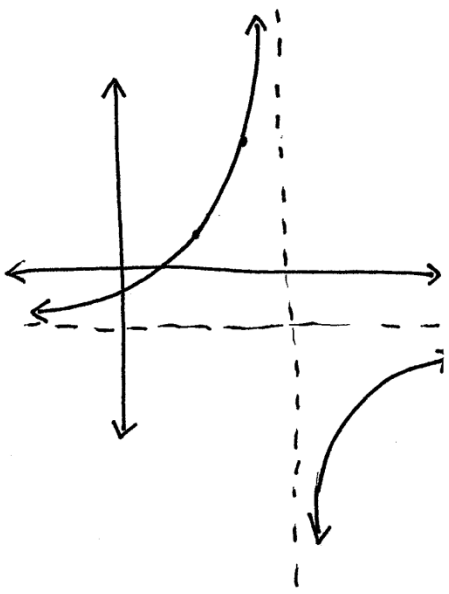
(17) $f(x) = \dfrac{-2x+3}{x-5}$

V.A : $x - 5 = 0$

$x = 5$

H.A : $y = -2$ (leading coeff rule)

x	3	4
y	1.5	5



(18) $f(x) = \dfrac{-x^2 - 3x}{x^2 + 2x - 3} = \dfrac{-x(x+3)}{(x+3)(x-1)} = \dfrac{-x}{x-1}$

VA: $x - 1 = 0$

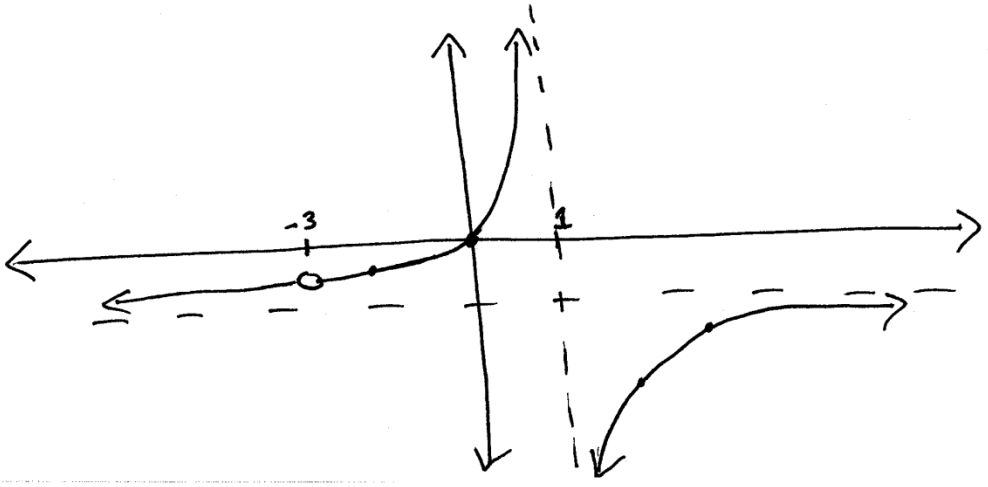
$x = 1$

HA: $y = -1$ (leading coeff rule)

Hole : $x + 3 = 0$

$x = -3$

x	2	3
y	-2	-1.5



(19) $f(x) = \dfrac{2x^2 - x}{x^3 + x^2 - 12x} = \dfrac{x(2x-1)}{x(x^2+x-12)} = \dfrac{2x-1}{x^2+x-12}$

VA: $x^2 + x - 12 = 0$

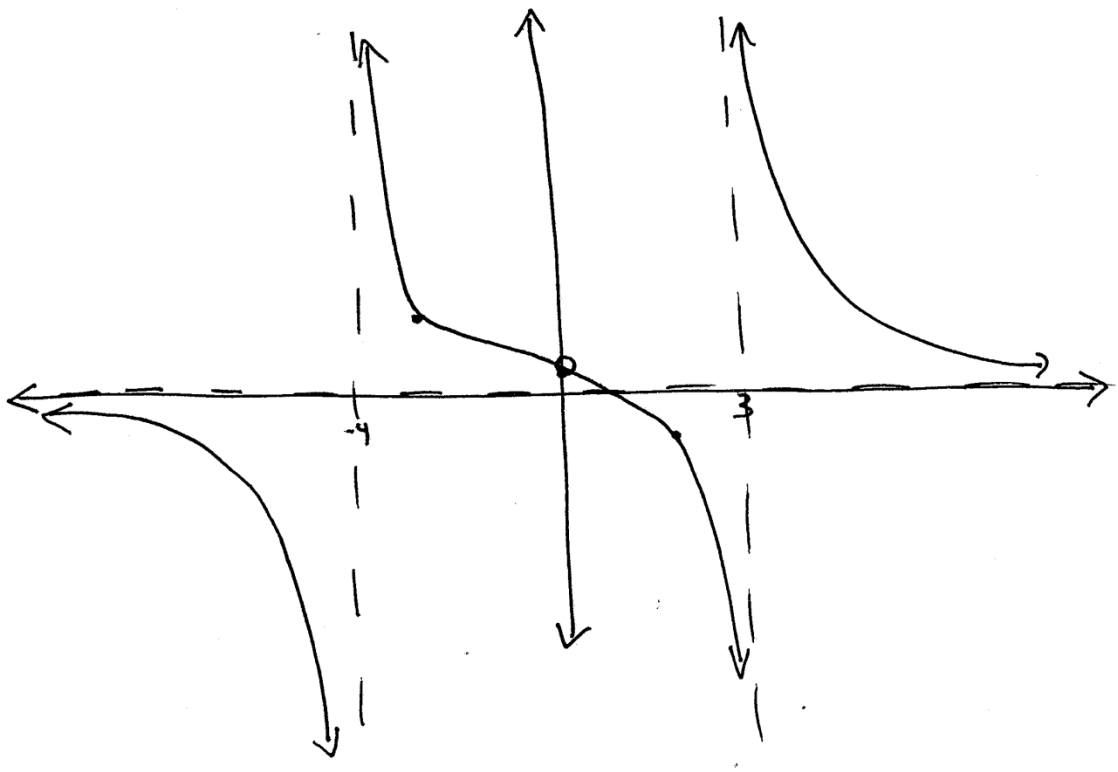
$(x+4)(x-3) = 0$

$x = -4, \; x = 3$

HA: $y = 0$ (Power of x is greater in denom)

Hole: $x = 0$

x	-3	0	2
y	$1.16\overline{6}$	$.08\overline{33}$	$-.5$



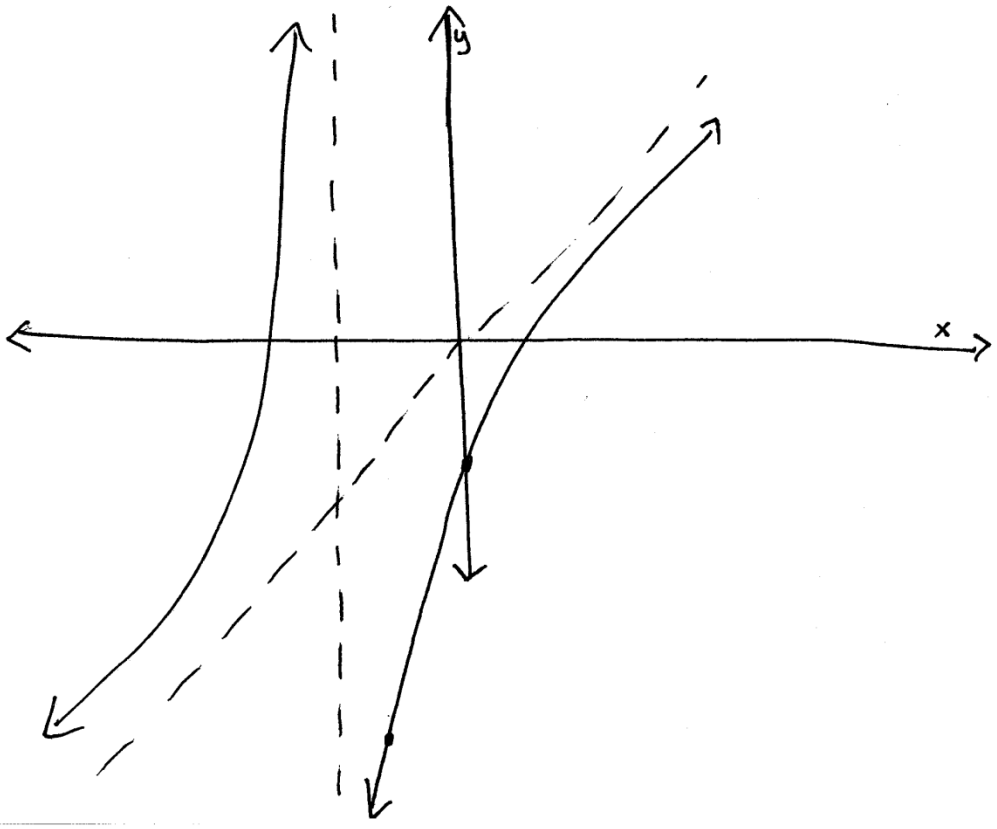
(20) $f(x) = \dfrac{x^2 + 3x - 28}{x + 3} = \dfrac{(x+7)(x-4)}{x+3}$ Does not simplify

V.A: $x + 3 = 0$, $-3 \quad -3$, $x = -3$

SA: $-3 \mid 1 \quad 3 \quad -28$, $1 \quad 0 \mid -28$

VA: $y = x$

x	-2	0
y	-30	$-9.3\overline{3}$



(21) $f(x) = \dfrac{3x^2 - 5}{x^2 + x - 20}$

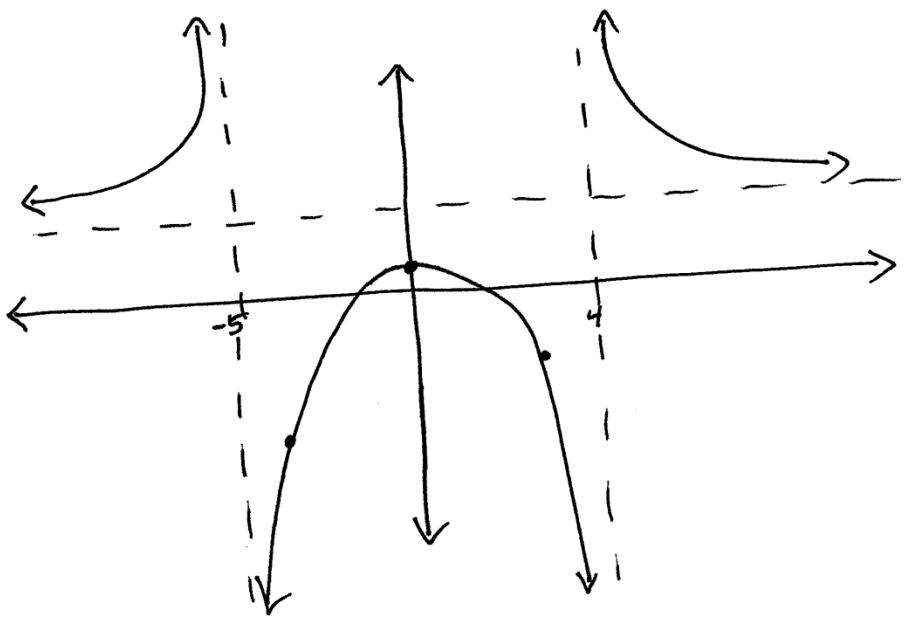
VA: $x^2 + x - 20 = 0$

$(x+5)(x-4) = 0$

$x = -5 \quad x = 4$

HA: $y = 3$
(leading coeff)

x	-4	0	3
y	-5.375	$.25$	-2.75



(22) $f(x) = \dfrac{x^3+2x^2-19x-20}{x^2-2x-8} = \dfrac{}{(x-4)(x+2)}$

*use Synthetic division to factor numerator

x	1	2	−19	−20
−2	1	0	−19	18
4	1	6	5	0

$(x-4)(x^2+6x+5)=0$

$(x-4)(x+5)(x+1)$

$f(x) = \dfrac{(x-4)(x+5)(x+1)}{(x-4)(x+2)}$

$f(x) = \dfrac{x^2+6x+5}{x+2}$

VA: $x+2=0$

$x=-2$

SA: $-2\,|\,1\ \ 6\ \ 5$

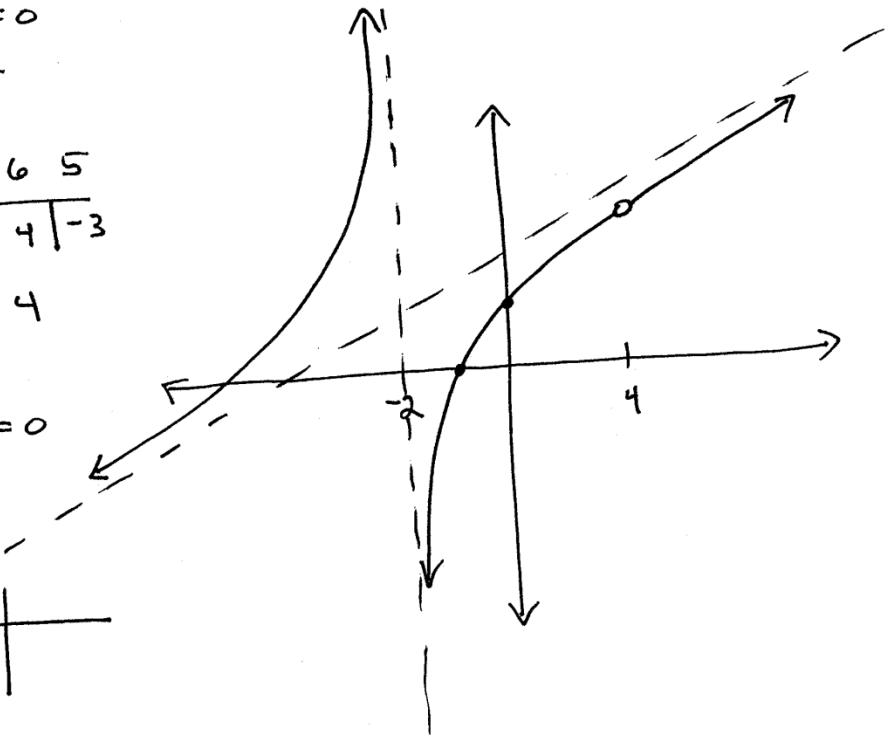
$\quad\ \ 1\ \ 4\ \ |-3$

$y = x+4$

Hole: $x-4=0$

$x=4$

x	−1	0	
y	0	2.5	



(23) $4x^2 + 7x - 4 \leq 3x^2 - 10$

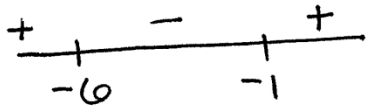
$x^2 + 7x + 6 \leq 0$

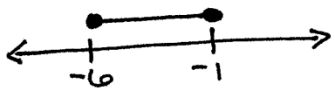
Critical Values:

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$x = -6 \quad x = -1$





$[-6, -1]$

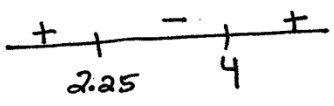
(24) $\frac{3x+2}{x-4} \geq -5 \rightarrow \frac{3x+2}{x-4} + \frac{5}{1}\cdot\frac{(x-4)}{(x-4)} \geq 0$

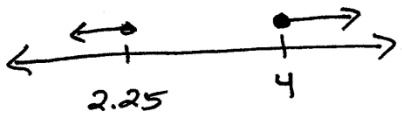
$\frac{8x-18}{x-4} \geq 0$

critical values:

$8x - 18 = 0 \quad x - 4 = 0$

$x = \frac{9}{4} = 2.25 \quad x = 4$





$(-\infty, 2.25] \cup [4, \infty)$

(25) $2x^3 + 7x^2 - 18x + 2 < 2x^2 + 47$

$2x^3 + 5x^2 - 18x - 45 < 0$

$P = \pm (1, 3, 5, 9, 15, 45)$

$q = \pm (1, 2)$

$PRR = \pm \left(1, 3, 5, 9, 15, 45, \frac{3}{2}, \frac{5}{2}, \frac{9}{2}, \frac{15}{2}, \frac{45}{2}\right)$

x	2	5	−18	−45
−1	2	3	−21	−24
−3	2	−1	−15	0

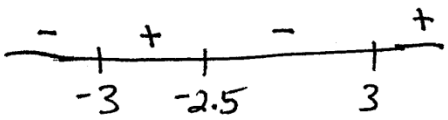
$f(x) = + \quad + \quad - \quad - \quad ①$

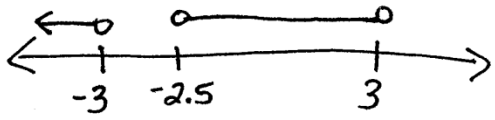
$f(-x) = - \quad + \quad + \quad - \quad ②$

$(x+3)(2x^2 - x - 15) = 0$

$(x+3)(2x+5)(x-3) = 0$

critical values: $x = -3, -2.5, 3$





$(-\infty, -3) \cup (-2.5, 3)$